MATH 112A Review: Geometric meaning of gradient and Linearization

Let $f(x, y) = xy^2$.

1. What is the gradient of f at the point $(0, 1)$?

> **Solution:** We have that $f_x = y^2$ and $f_y = 2xy$. Thus, $\nabla f(0, 1) = (1, 0)$.

2. What direction increases the fastest from the point $(0, 1)$ and what is the rate of change in the same direction?

> **Solution:** The direction that increases the fastest from the point $(0, 1)$ is the vector $(1, 0)$ with rate of change $||(1, 0)|| = 1$.

3. What is the linearization of f at the point $(0, 1)$?

> **Solution:** The linearization is
> $$\begin{aligned} L_f(x, y) &= f(0, 1) + \nabla f(0, 1) \cdot ((x, y) - (0, 1)) \\ &= 0 + (1, 0) \cdot ((x, y - 1)) \\ &= x. \end{aligned}$$